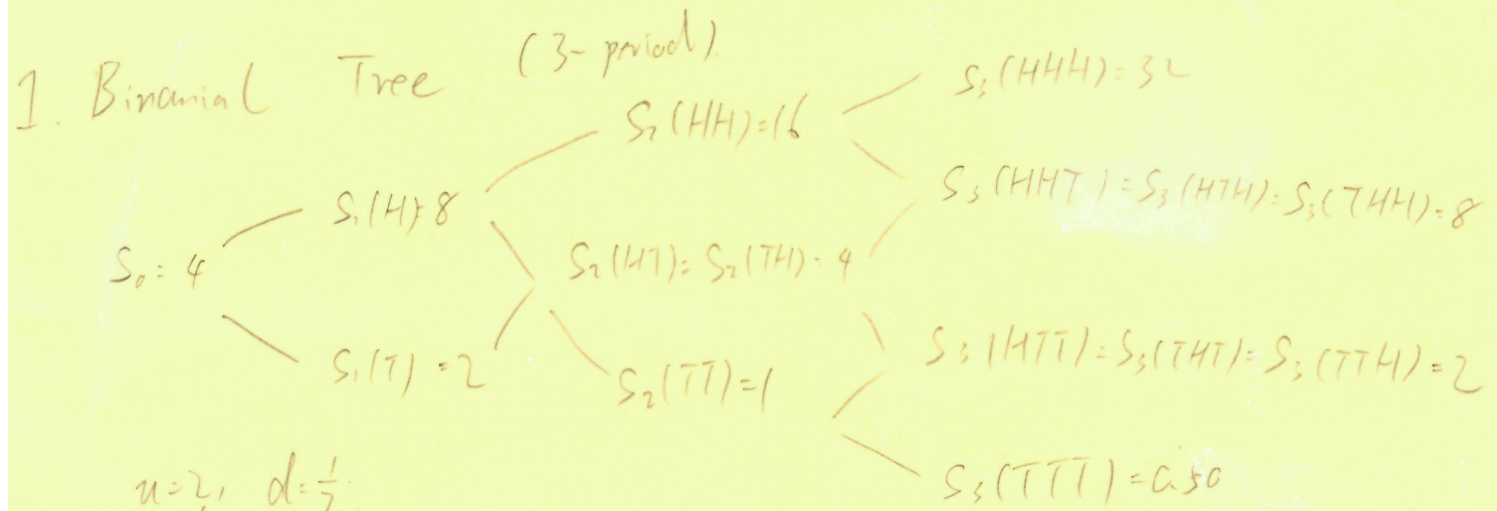


1.2.4

1.3.2.



2. Wealth Process.
Option

V_i : Value at time i .

Δ_i : share of stock at time i .

X_i : portfolio value at time i .

We make decision at time i , and see the result at time $i+1$.

$$X_1 = \Delta_0 S_1 + (1+r)(V_0 - \Delta_0 S_0) \quad \text{No arbitrage, } X_i = V_i$$

Next Step: $X_2 = \Delta_1 S_2 + (1+r)(X_1 - \Delta_1 S_1)$

$$X_{n+1} = \Delta_n S_{n+1} + (1+r)(X_n - \Delta_n S_n)$$

Since no arbitrage $X_i = V_i$.

2. Delta Hedging

Risk Neutral probability \tilde{p}, \tilde{q} , such that present value of \wedge ^{expectation} $i+1$ step is equal to i step $S_0 = \frac{1}{1+r} [\tilde{p} S_1(H) + \tilde{q} S_1(T)]$

$$V_n(w_1, \dots, w_n) = \frac{1}{1+r} [\tilde{p} V_{n+1}(w_1, w_2, \dots, w_n, H) + \tilde{q} V_{n+1}(w_1, w_2, \dots, w_n, T)]$$

$$\tilde{p} = \frac{1+r-d}{u-d}, \quad \tilde{q} = \frac{u-1-r}{u-d}$$

Solve the wealth process

$$\Delta_n(W_1, \dots, W_n) = \frac{V_{n+1}(W_1, \dots, W_{n+1}) - V_{n+1}(W_1, \dots, W_n T)}{S_{n+1}(W_1, \dots, W_{n+1}) - S_{n+1}(W_1, \dots, W_n T)}$$

Example 1: Lookback option

$$V_3 = \max_{0 \leq n \leq 3} S_n - S_3$$

$$\textcircled{1} V_0 = 1.376$$

Share option and get 1.376.

Buy stock 0.1733 shares.

The rest 0.6827 invests risk-free market.

Example 2:

Write down the pairs (S_t, M_t) , we have

v_3 (32, 32), (8, 16), (8, 8), (2, 8), (2, 4), (0.5, 4)

$\textcircled{2}$ 0, 8, 0, 6, 2, 3.5

v_2 (16, 16), (4, 8), (4, 4), (1, 4)

7.2, 2.4, 0.8, 2.2

v_1 (8, 8), (2, 4)

2.24, 1.2

v_0 (4, 4)

1.376