1.2.4

1.3.2.

1. Binomial Tree (3-period)

$S_0 = 4$

$S_1(H) = 8$

$S_1(T) = 2$

$S_2(HH) = 16$

$S_2(HT) = S_2(TH) = 4$

$S_2(TT) = 1$

$S_3(HHH) = 32$

$S_3(HHT) = S_3(HTH) = S_3(THH) = 8$

$S_3(HTT) = S_3(THT) = S_3(TTH) = 2$

$S_3(TTT) = 0.50$

$u = 2, \; d = \frac{1}{2}$

2. Wealth Process.

V_i: option Value at time i.

Δ_i: share of stock at time i.

X_i: portfolio value at time i.

We make decision at time i, and see the result at time $i+1$.

$X_1 = \Delta_0 S_1 + (1+r)(V_0 - \Delta_0 S_0)$ ~~No arbitrage, $X_i = V_i$~~

Next Step: $X_2 = \Delta_1 S_2 + (1+r)(X_1 - \Delta_1 S_1)$

$\vdots$

$X_{n+1} = \Delta_n S_{n+1} + (1+r)(X_n - \Delta_n S_n)$

Since no arbitrage $X_i = V_i$.

3. Delta Hedging

Explore Risk Neutral probability $\tilde{p}, \tilde{q}$, such that present value of expectation $i+1$ step is equal to i step $S_0 = \frac{1}{1+r}[\tilde{p} S_1(H) + \tilde{q} S_1(T)]$

$V_n(\omega_1, \cdots, \omega_n) = \frac{1}{1+r}[\tilde{p} V_{n+1}(\omega_1\omega_2\cdots\omega_n H) + \tilde{q} V_{n+1}(\omega_1\omega_2\cdots\omega_n T)]$

$\tilde{p} = \frac{1+r-d}{u-d}, \; \tilde{q} = \frac{u-1-r}{u-d}$

Solve the wealth process

$$\Delta_n(W_1,\cdots,W_n) = \frac{V_{n+1}(W_1,\cdots,W_nH) - V_{n+1}(W_1,\cdots,W_nT)}{S_{n+1}(W_1,\cdots W_nH) - S_{n+1}(W_1,\cdots W_nT)}$$

Example 1: Lookback option.

$$V_3 = \max_{0\le n\le 3} S_n - S_3$$

$V_0 = 1.376$

Short option and get 1.376.

Buy stock 0.1733 shares.

The rest 0.6827 invests risk-free market.

Example 2:

Write down the pairs (S_3, M_3), we have

V_3	(32, 32),	(8, 16)	(8, 8)	(2, 8)	(2, 4)	(0.5, 4)
V_3	0	8	0	6	2	3.5
V_2	(16, 16)	(4, 8)	(4, 4)	(1, 4)		
	3.2	2.4	0.8	2.2		
V_1	(8, 8)	(2, 4)				
	2.24	1.2				
V_0	(4, 4)					
	1.376					